# CHAOS CONTROL FOR TWO DISSIPATIVE SYSTEMS

## CONTROLUL HAOSULUI PENTRU DOUÃ SISTEME DISIPATIVE

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***Abstract.*** *Chaos is an interesting phenomenon closely related to nonlinear systems, which is very important for researchers in many disciplines. In medicine and chemistry the phenomenon of synchronization is of interest in studying cardiac rhythms and reactions in chemistry. In industry, synchronization is also used to ensure exact coincidence of frequencies in secure communications and in agriculture the problem of chaos control is important in biological control of weed populations and pests. In this work the master-slave, mutual synchronization and antisynchronization are used in order to control two chaotic dissipative systems governed by jerk functions. Our results show the transient time until synchronization depends on initial conditions of two systems and on the values of negative part of eigenvalues of control parameters. The synchronization and can be achieved for all methods but for amplification of chaos very closed initial conditions need to be chosen.*

**Key words**: nonlinear systems, dissipative flows, synchronization and antisynchronization

***Rezumat.*** *Controlul haosului este un fenomen interesant legat de sistemele neliniare, care este foarte important pentru cercetătorii din multe discipline. În medicina şi chimie fenomenul de sincronizare este de interes în studiul ritmului cardiac şi a reacţiilor chimice. În industrie sincronizarea este de asemenea utilizată pentru a asigura coincidenţa frecvenţelor necesară în securizarea comunicaţiilor iar în agricultură problema controlul haosului este important în controlul populaţiilor de buruieni şi dăunători. În această lucrare se foloseşte metoda de sincronizare master-slave, sincronizarea mutuală şi antisincronizarea a două sisteme haotice disipative. Rezultatele noastre arată că timpul de tranziţie până la sincronizare depinde de condiţiile inţtiale şi de partea negativă a valorilor proprii ale parametrilor de control. Sincronizarea este obţinută în toate cele trei metode dar pentru amplificare trebuie alese condiţii iniţiale foarte apropiate pentru cele două sisteme.*

**Cuvinte cheie**: sisteme neliniare, fluxuri disipative, sincronizare şi antisincronizare

## INTRODUCTION

Chaos is a very interesting phenomenon closely related to nonlinear systems, which is very important for workers in many disciplines (Mosekilde E. et al., 2002; Pikovsky A. et al., 2001). Chaotic behavior appearing in nonlinear systems has been received more attentions in the literature after Pecora and Carroll paper "Synchronization in chaotic systems" from 1990. Synchronization is a

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fundamental process in coupled dynamical systems. This means to design a controller or interconnections that guarantee synchronization of the multi-composed systems with respect to certain desired functional. In medicine and chemistry the phenomenon of synchronization is of interest in studying cardiac rhythms and reactions in chemistry; in industry, synchronization is also used to ensure exact coincidence of frequencies in secure communications.

Several methods of synchronization have been proposed and implemented. Jackson and Grosu (Jackson E. A and Grosu I., 1995; Grosu, I. ,1997) developed the open-plus-closed-loop (OPCL) method. This method gives precise driving for any continuous system in order to reach any desired dynamics and it has been applied to synchronization of two identical systems by Grosu (Grosu I., 2007) and Oancea (Oancea S., 2005). A similar strategy can be used for mutual synchronization (Lerescu A.I. et al., 2006). In addition Grosu and coworkers (Grosu I. et al., 2008) obtained the synchronization and antisynchronization in chaotic systems under parameter mismatch.

In this work the master-slave, mutual synchronization and antisynchronization are used in order to control two chaotic systems governed by jerk functions.

## THEORY

### 1. MASTER-SLAVE SYNCHRONIZATION

Let's consider a general master system:

$d\mathbf{X}/dt = \mathbf{F}(\mathbf{X});\ \mathbf{X} \in \mathbf{R}^n$ (1)

then the slave system:

$d\mathbf{x}/dt = \mathbf{F}(\mathbf{x}) + \mathbf{D}(\mathbf{x},\mathbf{X})$ (2)

where $\mathbf{D}(\mathbf{x},\mathbf{X}) = (\mathbf{A} - \partial \mathbf{F}/\partial \mathbf{x}|_{\mathbf{x}=\mathbf{X}})(\mathbf{x} - \mathbf{X}) - 1/2(\partial^2 \mathbf{F}/\partial \mathbf{x}^2)(\mathbf{x} - \mathbf{X})^2 - 1/6(\partial^3 \mathbf{F}/\partial \mathbf{x}^3)(\mathbf{x} - \mathbf{X})^3 + \ldots$

assures $\mathbf{x}(t) \rightarrow \mathbf{X}(t)$ for any $\| \mathbf{x}(0) - \mathbf{X}(0) \|$ small enough.

**A** is a constant Hurwitz matrix with negative real part eigenvalues. The matrix A should be chosen in such a manner in order that the coupling to be as simple as possible. This method was applied for Sprott,s collection of chaotic flows (Sprott J.C., 1997).

### 2. MUTUAL SYNCHRONIZATION

Let's consider two identical general oscillators:

$d\mathbf{x}/dt = \mathbf{F}(\mathbf{x});\ d\mathbf{y}/dt = \mathbf{F}(\mathbf{y});$ (3)

The coupled systems are:

$d\mathbf{x}/dt = \mathbf{F}(\mathbf{x}) + \mathbf{u}(\mathbf{x},\mathbf{y});\quad d\mathbf{y}/dt = \mathbf{F}(\mathbf{y}) + \mathbf{u}(\mathbf{x},\mathbf{y});$ (4)

where $\mathbf{u}(\mathbf{x},\mathbf{y}) = (A - d\mathbf{F}(\mathbf{s})/d\mathbf{s}) * (\mathbf{x}-\mathbf{y})/2$ , $\mathbf{s} = (\mathbf{x}+\mathbf{y})/2$ and A is the Hurwitz matrix.

The present method has been also applied to all systems from the Sprott collection

### 3. SYNCHRONIZATION AND ANTISYNCRONIZATION OF CHAOTIC SYSTEMS

Grosu and coworkers (Grosu et al., 2008) designed the coupling for stable synchronization and antisynchronization in chaotic systems under parameter mismatch.

The driver is:
d**y/**dt= **F**(**y**)+Δ**F(y);** (5)
**y** $\in$ **R**$^n$ **,** unde Δ**F(y)** contains mismatch parameters.
The driven system is given by:
d**x/**dt= **F**(**x**) **+D(x,**α**y)** (6)
where **D(x,**α**y)=α** d**y/**dt –**F**(α**y**)- (**A** -J **F**(α**y**)(**x** – α**y**)
J being the Iacobian and A the arbitrary constant Hurwitz matrix.

**4. JERK FUNCTIONS**

Gottlieb (Sprott J.C, 1997) showed that the simplest ODE in a single variable that can exhibit chaos is third order and he suggested for chaotic systems the form:

$$\dddot{x} = j(x, \dot{x}, \ddot{x})$$

where j is a jerk function (time derivative of acceleration).

Chaotic flows in three dimensions (3D) can be characterized as either dissipative or conservative, depending on the fractal dimension of the strange attractor. In this work we tried to synchronize two dissipative chaotic systems.

## RESULTS AND DISCUSSIONS

The simplest chaotic dissipative system was of the form (Sprott J.C, 1997):

$$\dddot{x} + A\ddot{x} - \dot{x}^2 + x = 0 \qquad (7)$$

where 2.017…<A<2.082… and which can be written on the form

$$\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= X_3 \qquad (8) \\ \dot{X}_3 &= -2.017X_3 + X_2^2 - X_1 \end{aligned}$$

The strange attractor of this system is given in fig.1

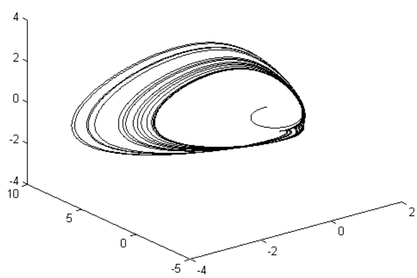


**Fig.1** – Phase portrait ($X_3$, $X_1$, $X_2$) for initial conditions [$X_1(0)=1$; $X_2(0)=0.1$;$X_3(0)=0.01$]

Choosing A=2.017 and the Hurvitz matrix having a single constant parameter, the Routh-Hurwitz conditions give for this parameter p, p<-5

With p=-10 the slave system is:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -2.017 x_3 + x_2^2 - x_1 + (-10 - 2X_2)(x_2 - X_2) - (x_2 - X_2)^2
\end{aligned}
\tag{9}
$$

Figures 2, 3 and 4 show the master-slave syncronization.

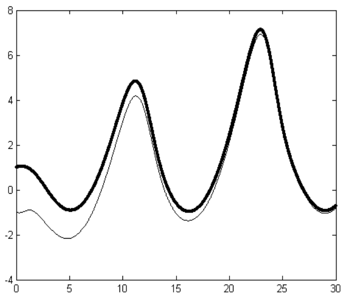


**Fig.2** – ($X_1(t)$ ▬, $x_1(t)$) -, for initial conditions [$X_1(0)=1$; $X_2(0)=0.1$;$X_3(0)=0.01$; $x_1(0)=-1$; $x_2(0)=-0.1$; $x_3(0)=-0.01$]

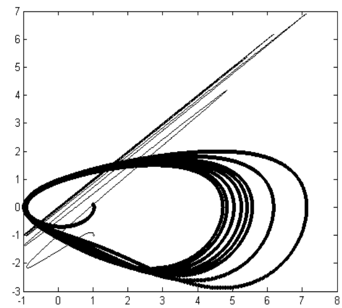


**Fig.3** – Phase portrait ($X_1$, $X_2$) ▬ and ($X_1$, $x_1$) - for the system (8) and (9) and initial conditions [$X_1(0)=1$; $X_2(0)=0.1$;$X_3(0)=0.01$; $x_1(0)=-1$; $x_2(0)=-0.1$; $x_3(0)=-0.01$]

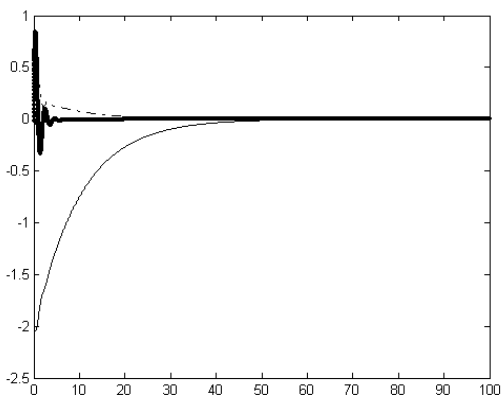


**Fig.4** – Synchronization errors for the systems (8)and (9); [($x_1(t) - X_1(t)$▬; $x_2(t) - X_2(t)$ .-; $x_3(t)$ - $X_3(t)$ -] for (13) and p=-10; [$X_1(0)=1$; $X_2(0)=0.1$;$X_3(0)=0.01$; $x_1(0)=-1$; $x_2(0)=-0.1$; $x_3(0)=-0.01$]

The mutual method of synchronization for this dissipative system gives:

$$\dot{x}_1 = x_2 \qquad \dot{y}_1 = y_2$$
$$\dot{x}_2 = x_3 \qquad \dot{y}_2 = y_3$$
$$\dot{x}_3 = -2.017x_3 + x_2^2 - x_1 + \qquad \dot{y}_3 = -2.017y_3 + y_2^2 - y_1 +$$
$$(-10-(x_2+y_2))(x_2-y_2)/2 \qquad (-10-(x_2+y_2))(-x_2+y_2)/2$$
(10)

Figures 5 and 6 show the mutual syncronization

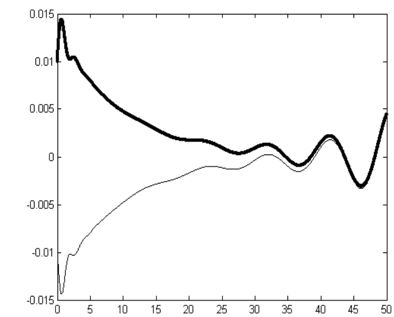

**Fig. 5** – ($x_1(t)$ -, $y_1(t)$) -, for p=-10 and initial conditions [$x_1(0)=x_2(0)=x_3(0)=0.01$; $y_1(0)=y_2(0)=y_3(0)=-0.01$]

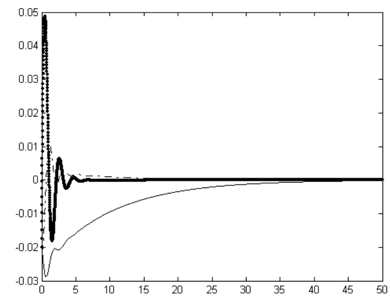

**Fig. 6** – Synchronization errors for the system (10); [($y_1(t) - x_1(t)$ -; $y_2(t) - x_2(t)$ .-; $y_3(t)$ - $x_3(t)$ -)] for and p=-10; [$x_1(0)=x_2(0)=x_3(0)=0.01$; $y_1(0)=y_2(0)=y_3(0)=-0.01$]

Amplification of chaos for the system (8) is given in the following equations

$$\dot{y}_1 = y_2 \qquad \dot{x}_1 = x_2$$
$$\dot{y}_2 = y_3 \qquad \dot{x}_2 = x_3$$
$$\dot{y}_3 = -2.017y_3 + y_2^2 - y_1 - 0.2y_3 \qquad \dot{x}_3 = -2.017x_3 + x_2^2 - x_1 + 0.4y_3 - 6y_2^2 +$$
$$(-10+4y_2)(x_2+2y_2)$$
(11)

Figures 7 and 8 show the antisyncronization of these systems.

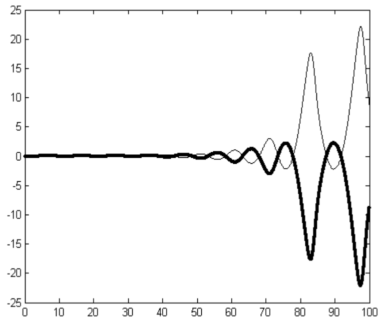


**Fig. 7** – (2$y_1$(t) -, $x_1$(t) -, for p=-10 and initial conditions [$x_1$(0)=$x_2$(0)=$x_3$(0)=0.01; $y_1$(0)=$y_2$(0)=$y_3$(0)=-0.01]

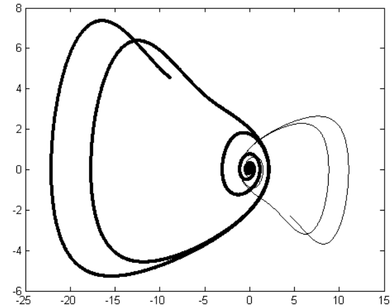


**Fig. 8** – Phase portrait ($y_1$, $y_2$)- and ($x_1$, $x_2$)- for the system (11) and initial conditions [$y_1$(0)=$y_2$(0)=$y_3$(0)=0.001, $x_1$(0)=$x_2$(0)=$x_3$(0)=-0.001]

# CONCLUSIONS

In this paper we presented master-slave, mutual synchronization and antisynchronization for chaotic systems governed by jerk functions. Our results show that the synchronization and can be achieved for all methods. The transient time until synchronization depends on initial conditions of two systems and on the values of negative part of eigenvalues. The methods presented in this work can be useful to be applied to other chaotic systems and can be generalized in order to find the main applications in biology, physics and industry.

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